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Implicit Differentiation

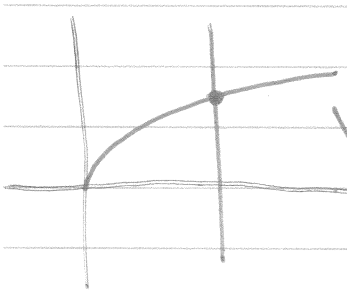
Explicit functions

Ex. $y = \sqrt{x}$

$y = \dfrac{x}{x^2+1}$

$\underline{y = f(x)}$

y in terms of x



Vertical line test ✓

$y^2 = 9 - x^2$

$y = \pm\sqrt{9-x^2}$

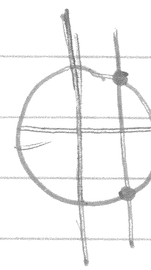
Implicit Functions

Ex. $x^2 + y^2 = 9$

$y^4 + y = x^3 - 3x$

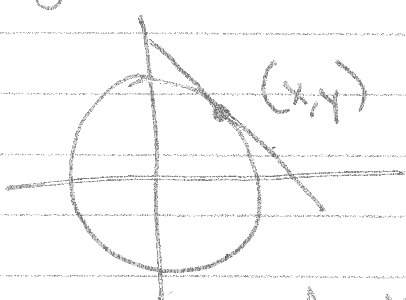
$g(x,y) = 0$

relationship b/w x & y



Vertical line test X

Q: $g(x,y)=0$. and (x,y) is a point on the graph,



Can we find the slope of the tangent line?

A: Yes!

Ex: $x^2 + y^2 = 9$

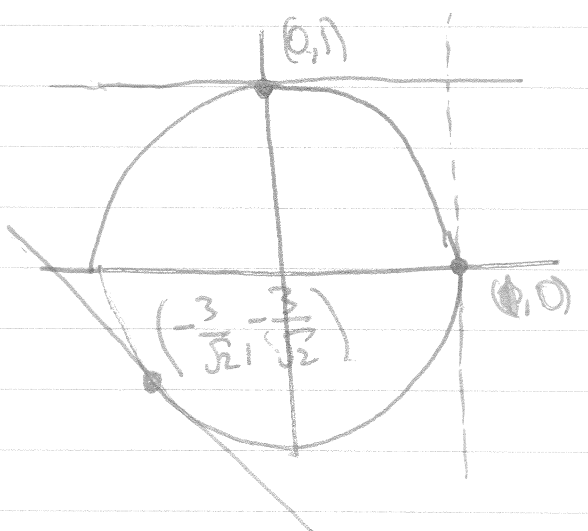
· Take $\frac{d}{dx}$ on both sides

$$\Big\downarrow \frac{d}{dx}$$

$$2x + 2y\frac{dy}{dx} = 0$$

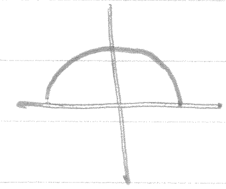
· Isolate $\frac{dy}{dx}$

$$2y\frac{dy}{dx} = -2x \rightsquigarrow \boxed{\frac{dy}{dx} = -\frac{x}{y}}$$



$$-\frac{-3/\sqrt{2}}{-3/\sqrt{2}} = -1$$

If you want, check $y = \sqrt{9-x^2}$.



$$\frac{dy}{dx} = \frac{1}{2\sqrt{9-x^2}}(-2x) = -\frac{x}{\sqrt{9-x^2}}$$

Ex. $y^4+y=x^3-3x$

$$\frac{dy}{dx}$$
$$y' = \frac{3x^2-3}{4y^3+1}$$

$$4y^3\frac{dy}{dx}+\frac{dy}{dx}=3x^2-3$$

$\frac{dy}{dx}$ = "rate of change of y with respect to x"

y' = "rate of change of y"

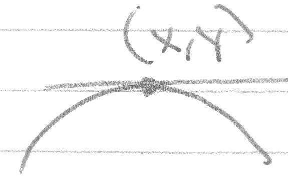
$$\boxed{\frac{dy}{dx}=\frac{3x^2-3}{4y^3+1}}$$

Ex. $x^2+3y^2-xy=11$

Q: Find the maximal possible value of y.
(find the highest point on the graph.)

Note: At the maximal point (x,y),

$$\frac{dy}{dx}=0$$



So: • Find $\frac{dy}{dx}$ • Set $\frac{dy}{dx}=0$ & solve

$$x^2 + 3y^2 - x \cdot y = 11$$

• $\frac{d}{dx}$ both sides

$$2x + 6y\frac{dy}{dx} - x \cdot \frac{dy}{dx} - 1 \cdot y = 0$$

• Solve for $\frac{dy}{dx}$

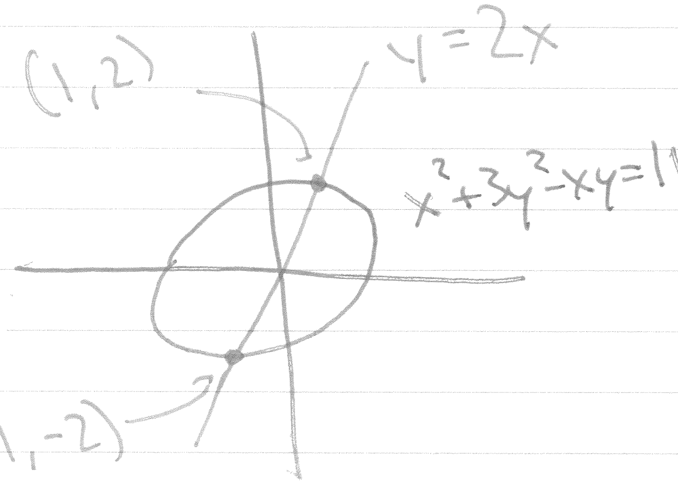
$$(6y - x)\frac{dy}{dx} = y - 2x$$

$$\boxed{\frac{dy}{dx} = \frac{y-2x}{6y-x}}$$

• Find the critical points

$$\frac{dy}{dx} = 0 \rightsquigarrow y - 2x = 0$$

$$\rightsquigarrow \boxed{y = 2x}$$



$$x^2 + 3(2x)^2 - x(2x) = 11$$

$$x^2 + 12x^2 - 2x^2 = 11$$

$$11x^2 = 11$$

$$x^2 = 1 \rightsquigarrow x = 1, -1$$

$$y = 2, -2$$

• Plug $y = 2x$ into $x^2 + 3y^2 - xy = 11$